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# Hydrogen Bonding in the Crystalline State. $\mathrm{CaHPO}_{4}$ (Monetite), $\boldsymbol{P} \overline{\mathrm{I}}$ or $\boldsymbol{P 1 ?}$ A Novel Neutron Diffraction Study* 

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1749 independent reflexions, measured on a single-crystal neutron diffractometer ( $\lambda=1.217 \AA$ ), were used for refining the crystal structure of $\mathrm{CaHPO}_{4}$ in both space groups $P \overline{1}(R=0.0283)$ and $P 1(R=0.0286)$. In the $P \overline{1}$ model a proton lies on a symmetry centre forming a symmetric hydrogen bond ( $2.459 \AA$ ) and another is statistically distributed between two centrosymmetric positions; in the $P 1$ models the first proton is slightly off the centre and the second is either ordered or disordered. By crystal-chemical considerations on the refinements, and taking into account a heat capacity anomaly at about 280 K together with very weak effects of piezoelectricity and of generation of the second optical harmonic at room temperature, the existence of a low-temperature ordered ( $P 1$ ) phase and a high-temperature disordered ( $P \overline{1}$ ) phase is suggested; diffraction data would come from a crystal in the disordered phase with a small fraction of the ordered one. A peculiar version of the KKM effect is shown by the difference maps. X-ray and neutron diffraction refinements are compared statistically.

## Introduction

$\mathrm{CaHPO}_{4}$ (monetite) has been the subject of several crystallographic (MacLennan \& Beevers, 1955; Jones \& Cruickshank, 1961; Curry, Denne \& Jones, 1968; Denne \& Jones, 1971; Dickens, Bowen \& Brown, 1972), spectroscopic (Blinc \& Hadži, 1960; Petrov, Soptrajanov, Fuson \& Lawson, 1967) and other physicochemical studies (Egan \& Wakefield, 1964).

The structural model refined in space group $P \overline{1}(Z=$ 4) by Dickens et al. (1972), with X-ray data ( $R=$ 0.031 ), implies that $\mathrm{H}(1) \dagger$ is on an inversion centre, forming a symmetric hydrogen bond $\mathrm{O}(7) \cdots \mathrm{O}\left(7^{\prime}\right)=$

[^0]2.458 (2) $\AA$ [type $A$ (Currie \& Speakman, 1970) or symmetry-restricted (SR) hydrogen bond (Catti \& Ferraris, 1974)]; $\mathrm{H}(2)$ forms the hydrogen bond $\mathrm{O}(1) \cdots \mathrm{O}\left(5^{\prime}\right)^{\mathrm{i}}=2.565(1) \AA$, and the third independent hydrogen atom, $\mathrm{H}(3)$, is disordered over two centrosymmetric positions and is involved in the hydrogen bond $\mathrm{O}(6) \cdots \mathrm{O}\left(8^{\prime}\right)^{i i}=2 \cdot 669$ (1) $\AA$ which is also an edge of the coordination polyhedron of $\mathrm{Ca}\left(2^{\prime}\right)$ (Fig. 1). [101] chains of $\mathrm{P}(2)$ coordination tetrahedra, linked via the hydrogen bonds involving $\mathrm{H}(1)$ and $\mathrm{H}(3)$ are present in the structure; the $\mathrm{P}(1)$ tetrahedra are connected to these chains by the third hydrogen bond.

However, according to Denne \& Jones (1971) 'Professor W. G. Perdok found that monetite gave the weakest positive piezoelectric effect yet detected with his Giebe \& Scheibe apparatus'; such an effect was also reported by the same authors for the isostructural (Ferraris \& Chiari, 1970) $\mathrm{CaHAsO}_{4}$ (weilite). At that time, they proposed a $P 1$ model with $\mathrm{H}(3)$ ordered and $\mathrm{H}(1)$ in an asymmetric position; however, the quality and quantity of their neutron diffraction data did not allow


Fig. 1. Projection of the $P \overline{1}$ structure of $\mathrm{CaHPO}_{4}$ onto the plane normal to $\mathbf{b}$, emphasizing the hydrogen-bonding scheme and the $\mathrm{PO}_{4}$ tetrahedra pattern ( P atoms are not shown).
them to choose a definite ordered model (presence of $\mathrm{HPO}_{4}$ groups alone or together with $\mathrm{PO}_{4}$ and $\mathrm{H}_{2} \mathrm{PO}_{4}$ groups) and to remove the heavy atoms from centrosymmetric positions. The last point is clearly a serious crystal-chemical drawback, particularly as far as the O atoms bonded to $\mathrm{H}(1)$ and $\mathrm{H}(3)$ are concerned.

## Experimental

A sample of synthetic $\mathrm{CaHPO}_{4}$ prepared by Dr V. R. Lehr (TVA, Alabama) was supplied by Dr D. W. Jones (Bradford University); a single crystal was ground to a rough cylinder $11 \mathrm{~mm}^{3}$ (diameter $1.7-1.9 \mathrm{~mm}$, length $4.3 \mathrm{~mm}),[010]$ elongated within a few degrees.

Neutron diffraction intensities were collected on the D8 four-circle diffractometer at the HFR of the Institute Laue-Langevin (Grenoble). The following measurement conditions were used: room temperature; $\lambda=1.217 \AA$ with $\mathrm{Cu}(200)$ monochromator; neutron beam divergence $60^{\prime}$; beam flux onto the crystal $10^{7} \mathrm{n}$ $\mathrm{cm}^{-2} \mathrm{~s}^{-1}$; step-scan technique $(\omega-2 \theta)$ with 31 steps for every reflexion; $2 \theta \leq 120^{\circ}$; scan width and pre-set monitor counting increasing with $2 \theta$ ( 2.50 to $6.15^{\circ}$ in $2 \theta$ and up to $50 \%$ counting increase); the 341 and 501 reflexions were measured as references every 40 reflexions. Probably because of a fracture on the surface of the monochromator, the peak profiles looked asymmetric with an inflexion on the low- $\theta$ side. For each reflexion, the counts for the first three and last three steps were averaged to obtain the background, with a program written by one of us (AF); after removal of 140 reflexions with $I \leq 3 \sigma(I)$, a set of 1749 independent observations was left.

The cell parameters of Dickens et al. (1972) were used: $a=6.910(1), b=6.627(2), c=6.998$ (2) $\AA$, $\alpha=96.34$ (2),$\quad \beta=103.82$ (2), $\gamma=88.33(2)^{\circ}$. The scattering lengths were (Bacon, 1972): $\mathrm{P}, 5 \cdot 10 ; \mathrm{Ca}$, $4.70 ; \mathrm{O}, 5.80$ and $\mathrm{H},-3.74 \times 10^{-13} \mathrm{~cm}$.

The absorption correction was neglected (measured absorption coefficient: $\mu=1.2 \mathrm{~cm}^{-1}$ ). All the main calculations were performed at the Computing Centre of the University of Torino with the programs of the XRAY system (Stewart, Kundell \& Baldwin, 1970) and with a modified version of ORFLS (Busing, Martin \& Levy, 1962).

## Refinement

## P1 model

A difference map following a structure factor calculation with the X-ray heavy-atom parameters (Dickens et al., 1972) showed $\mathrm{H}(1)$ on the symmetry centre, $\mathrm{H}(2)$ in a general position and bonded to $\mathrm{O}(1)$, and $H(3)$ statistically bonded to both $O(6)$ and $O\left(6^{\prime}\right)$. However, the $\mathrm{H}(3)$ peak $\left(-51 \times 10^{-14} \mathrm{~cm} \AA^{-3}\right)$ was less than $\frac{1}{3}$ of those corresponding to $\mathrm{H}(1)$ and $\mathrm{H}(2)(-168$ and $-183 \times 10^{-14} \mathrm{~cm} \AA^{-3}$ ) respectively. The isotropic ( $R=0.090$ ) and anisotropic ( $R=0.046$ ) full-matrix least-squares refinements converged rapidly. A correction for secondary extinction by least-squares interpolation of the function $I_{c} / I_{o}=k\left(1+g I_{c}\right)$ yielded $k=$ 0.900 and $g=32 \times 10^{20} \mathrm{~cm}^{-2}$; the refinement converged then to $R=0.0283$. Unit weights were used because of the satisfactory constant value of $\Delta^{2}$ averaged over groups vs $\left|F_{o}\right|$ and $\sin \theta$. Very weak reflexions showed $\left|F_{o}\right|>\left|F_{c}\right|$ almost systematically, presumably because of the multiple scattering contribution to $\left|F_{o}\right|$; no correction was made for thermal diffuse scattering.

## P1 models

The full-matrix least-squares refinements in $P 1$ were performed by minimizing the quantity $\Sigma_{i} w_{i}\left|F_{o}-F_{c}\right|_{i}^{2}$ instead of $\Sigma_{i} w\left(\left|F_{o}\right|-\left|F_{c}\right|\right)_{i}^{2}$ (Rae, 1974). The high covariance between parameters in pseudo-symmetric structures, or even singularities when parameters are exactly symmetry-related, is avoided by this method.

A P1 model for $\mathrm{CaHPO}_{4}$ can be obtained (Denne \& Jones, 1971) by ordering $\mathrm{H}(3)$, so that the inversion centre must disappear; the whole $\mathrm{H}(3)$ atom was then attached to $O(6)$, and a subsequent Fourier map showed $H(1)$ very slightly closer to $O\left(7^{\prime}\right)$. The refinement converged ( $R=0.080$ and $R=0.042$ for the isotropic and anisotropic cases respectively) with $\mathrm{H}(1)$ linked to $O\left(7^{\prime}\right) ; O(8)$ and $O\left(8^{\prime}\right)$ were appreciably noncentrosymmetric, but the other atoms practically centrosymmetric; the position of $\mathrm{P}(1)$ was kept fixed in order to define the origin. At this stage the O atoms
related to $\mathrm{H}(3)$ and $\mathrm{H}(1)$ [i.e. $\mathrm{O}(6), \mathrm{O}\left(6^{\prime}\right)$ and $\mathrm{O}(7)$, $\left.O\left(7^{\prime}\right)\right]$ were given shifts along their respective lines of maximum thermal motion, so as to lengthen the $\mathrm{P}-\mathrm{OH}$ bond with respect to $\mathrm{P}-\mathrm{O}$; the refinement was continued without substantial improvement of $R$ which, however, dropped to 0.0286 after a secondary extinction correction ( $k=0.933, g=21 \times 10^{20} \mathrm{~cm}^{-2}$ ).

Refinements were tried for two different models, and $R$ did not change significantly: (i) $\mathrm{H}(1)$ and $\mathrm{H}(3)$ bonded to the same tetrahedron - appreciable shifts of the O atoms could not be observed; (ii) $\mathrm{H}(1)$ bonded to $\mathrm{O}\left(7^{\prime}\right)$ and $\mathrm{H}(3)$ disordered over $\mathrm{O}(6)$ and $\mathrm{O}\left(6^{\prime}\right)$ refining the occupancy factors of $H(3)$ and $H\left(3^{\prime}\right)$ showed only a slight preference, about $4 \%$, for the half-atom bonded to $\mathrm{O}(6)$.

The final fractional coordinates and the $B_{i j}$ thermal coefficients of the expression

$$
\exp \left(-\frac{1}{4} \sum_{i=1}^{3} \sum_{j=1}^{3} B_{i j} h_{i} h_{j} a^{*} a_{j}^{*}\right)
$$

are given in Table 1 for both the $P \overline{1}$ and the $P 1$ refinements. $\dagger P 1$ parameters are reported only for the ordered case with $\mathrm{H}(1)$ and $\mathrm{H}(3)$ attached to different $\mathrm{PO}_{4}$ tetrahedra, as it seems to be the simplest and extreme model.
$\dagger$ A list of $\left|F_{o}\right|$ and $F_{c}$ (for P $\overline{1}$ only) has been deposited with the British Library Lending Division as Supplementary Publication No. SUP 32206 ( 9 pp .). Copies may be obtained through The Executive Secretary, International Union of Crystallography, 13 White Friars, Chester CH1 1NZ, England.

Table 1. Fractional coordinates $\left(\times 10^{4}\right)$ and vibrational parameters $\left(\AA^{2} \times 10^{2}\right)$ for the $P \overline{1}($ first line $)$ and the $P 1$ (second and third lines) models
The e.s.d.'s ( $P 1$ in parentheses) are as follows for positional and thermal parameters, in the order: 0.0001 ( 0.0002 ), 0.02 ( 0.04 ) for $\mathrm{Ca}, \mathrm{P}$ and $\mathrm{O}: 0(0.0004), 0.08(0.08)$ for $\mathrm{H}(1) ; 0.0002(0.0004), 0.05(0.08)$ for $\mathrm{H}(2) ; 0.0008(0.0008), 0.22(0.20)$ for $\mathrm{H}(3)$.

|  | $x$ | $y$ | $z$ | $B_{11}$ | $B_{22}$ | $B_{33}$ | $B_{12}$ | $B_{13}$ | $B_{23}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(1)$ | 2077 | 3790 | 7207 | 43 | 47 | 52 | 1 | 16 | 4 |
|  | 2077 | 3790 | 7207 | 40 | 44 | 47 | -1 | 15 | -1 |
|  | -2077 | -3791 | -7200 | 39 | 40 | 51 | 2 | 16 | 9 |
| $\mathbf{P}$ (2) | 2962 | 9436 | 2082 | 75 | 76 | 62 | 18 | 17 | 2 |
|  | 2955 | 9425 | 2089 | 79 | 69 | 61 | 19 | 19 | -4 |
|  | -2971 | -9452 | -2076 | 71 | 75 | 57 | 18 | 14 | 5 |
| $\mathrm{Ca}(1)$ | 2947 | 4353 | 2719 | 62 | 120 | 95 | 3 | 25 | -26 |
|  | 2925 | 4369 | 2719 | 60 | 113 | 88 | 5 | 20 | -23 |
|  | -2968 | -4337 | -2717 | 55 | 112 | 95 | 3 | 23 | -30 |
| $\mathrm{Ca}(2)$ | 1754 | 8373 | 6664 | 114 | 56 | 91 | -4 | 3 | 17 |
|  | 1762 | 8370 | 6662 | 103 | 52 | 90 | -5 | 0 | 11 |
|  | -1745 | -8375 | -6657 | 110 | 53 | 87 | -2 | 2 | 19 |
| $\mathrm{O}(1)$ | 3234 | 3322 | 9373 | 100 | 119 | 55 | 19 | 19 | 22 |
|  | 3228 | 3335 | 9385 | 98 | 108 | 53 | 22 | 20 | 22 |
|  | -3239 | -3309 | -9358 | 97 | 118 | 53 | 18 | 15 | 17 |
| O (2) | 3514 | 4919 | 6324 | 65 | 90 | 82 | -14 | 28 | 24 |
|  | 3503 | 4915 | 6328 | 64 | 86 | 83 | -11 | 31 | 28 |
|  | -3523 | -4925 | -6321 | 61 | 86 | 80 | -16 | 29 | 19 |
| O(3) | 1390 | 1806 | 5964 | 94 | 56 | 97 | -8 | 12 | -14 |
|  | 1377 | 1793 | 6006 | 93 | 50 | 91 | -10 | 16 | -11 |
|  | $-1403$ | -1820 | -5919 | 91 | 55 | 91 | -5 | 11 | -14 |
| O(4) | 394 | 5228 | 7459 | 58 | 76 | 131 | 16 | 32 | 2 |
|  | 388 | 5224 | 7491 | 57 | 71 | 126 | 16 | 30 | 3 |
|  | -402 | -5234 | -7428 | 50 | 71 | 124 | 17 | 29 | 2 |
| O(5) | 3329 | 8363 | 168 | 95 | 179 | 75 | 35 | 14 | -37 |
|  | 3348 | 8406 | 164 | 92 | 158 | 71 | 29 | 20 | -35 |
|  | -3307 | -8315 | -174 | 86 | 167 | 73 | 32 | 14 | -36 |
| O(6) | 4592 | 1042 | 3007 | 90 | 134 | 176 | -35 | 13 | -52 |
|  | 4589 | 1018 | 3069 | 85 | 135 | 167 | -38 | 16 | -55 |
|  | -4598 | -1067 | -2946 | 85 | 125 | 161 | -32 | 6 | -40 |
| O(7) | 994 | 673 | 1637 | 83 | 136 | 79 | 48 | 16 | 3 |
|  | 987 | 628 | 1666 | 82 | 124 | 74 | 52 | 18 | 6 |
|  | -1001 | -721 | -1608 | 77 | 124 | 78 | 47 | 15 | 4 |
| $\mathrm{O}(8)$ | 2880 | 7917 | 3537 | 230 | 125 | 159 | 74 | 113 | 85 |
|  | 2779 | 7863 | 3475 | 191 | 115 | 123 | 53 | 79 | 61 |
|  | -2977 | -7972 | -3598 | 186 | 111 | 127 | 55 | 78 | 75 |
| H(1) | 0 | 0 | 0 | 189 | 261 | 337 | 70 | 115 | 67 |
|  | -34 | 16 | -76 | 182 | 251 | 305 | 73 | 113 | 73 |
| H(2) | 4605 | 2588 | 9506 | 233 | 236 | 144 | 3 | 42 | 16 |
|  | 4615 | 2580 | 9537 | 238 | 229 | 142 | 6 | 51 | 18 |
|  | 5406 | 7407 | 522 | 224 | 241 | 143 | 6 | 41 | 53 |
| H(3) | 5293 | 1160 | 4683 | 515 | 653 | 467 | 320 | 328 | 274 |
|  | 5278 | 1164 | 4677 | 489 | 644 | 461 | 292 | 319 | 238 |

## Comparison between the refinements

In view of the reasonable equal $R$ values for both refinements, and taking into account that the number of variables is nearly doubled in changing from $P \overline{1}$ (133) to $P 1$ (262), the centrosymmetric model seems to be supported by the diffraction experiment; we can, however, emphasize and discuss some further features. (i) The lower $R$ for $P 1$ before the secondary extinction correction, compared to the equal final values for $P 1$ and $P \overline{1}$, can be interpreted by assuming that such a correction is taken into account, in part, by extra degrees of freedom in $P 1$. (ii) The $P 1$ refinement should be favoured by the better $F_{o} / F_{c}$ agreement for weak reflexions (Rae, 1975), but this effect may also be due to the higher number of variables. (iii) Thermal factors of the atoms are not very different in the two models, except for $\mathrm{O}(8)$, which also shows the greatest positional shift: however, higher anisotropy of thermal motion in the coordination tetrahedron of $\mathrm{P}(2)$, which is common to both $P \overline{1}$ and $P 1$ models, would support a disordered model since only the atoms close to $\mathrm{H}(3)$ are involved. (iv) Although average positions of pseudocentrosymmetric pairs of atoms in $P 1$ and corresponding atomic positions in P1 are not significantly different, single coordinates differ by several e.s.d.'s from the average value. Important differences in bond distances and angles may therefore arise, whose crystal-chemical meaning should be crucial in choosing the model.

## Discussion

## Crystal-chemical considerations

The disordered $P \overline{1}$ model must represent an average image, such as is revealed by diffraction effects, over two structural configurations that are distinct in neighbouring cells or small groups of cells. The disorder is assumed to be static, because $\mathrm{H}(3) \cdots \mathrm{H}\left(3^{\prime}\right)=1.7 \AA$ seems to be too long for dynamic effects. The two configurations should respectively show $\mathrm{H}(3)$ bonded to $\mathrm{O}(6)$ only and to $\mathrm{O}\left(6^{\prime}\right)$ only: other atomic shifts would adjust distances and angles according to the position of $\mathrm{H}(3)$ : in particular, in each configuration the two tetrahedra of $P(2)$ and $P\left(2^{\prime}\right)$ are chemically inequivalent, so that $\mathrm{H}(1)$ links to one of them preferentially and the symmetric hydrogen bond disappears (Catti \& Ferraris, 1976). In this context $\mathrm{O}(6)-\mathrm{H}(3)=1.153 \AA$, which is remarkably long for a hydrogen bridge $\mathrm{O} \cdots \mathrm{O}$ of 2.658 (1) A A , can be understood.

The crystal-chemical consistency of the ordered $P 1$ model with $\mathrm{H}(3)$ linked to $\mathrm{O}(6)$ can be tested by comparing pseudocentrosymmetric bond lengths directly involved with $\mathrm{H}(1)$ and $\mathrm{H}(3)$ (Tables 2, 3, 4). The two bonds $\mathrm{P}(2)-\mathrm{O}(6)^{\text {iv }}$ and $\mathrm{P}\left(2^{\prime}\right)-\mathrm{O}\left(6^{\prime}\right)^{\text {vi }}$, approximately equal, are the weak point of this model; the former would have been expected to be longer than the latter. By contrast, a satisfactory behaviour is shown by other pairs of pseudosymmetric bonds, where significant differences of length can be observed; besides, $\mathrm{O}(6)-$

Table 2. Interatomic distances and $\mathrm{O}-\mathrm{P}-\mathrm{O}$ angles for the P tetrahedra
Under $P 1$ the two columns refer to the polyhedron reported for $P \overline{1}$ and to its pseudo-centrosymmetric polyhedron. The e.s.d.'s are $0.001 \AA(0.002 \AA)$ and $0.1^{\circ}$ for distances ( $P 1$ in parentheses) and angles respectively.

|  | $P \overline{1}$ |  | $P 1$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(1)-\mathrm{O}(1)$ | 1.595 | $\square$ | $1.599 \AA$ |  | $1.591 \AA$ |  |
| $\mathrm{P}(1)-\mathrm{O}(2)$ | 1.538 |  | 1.530 |  | 1.546 |  |
| $\mathrm{P}(1)-\mathrm{O}(3)$ | 1.516 |  | 1.514 |  | 1.520 |  |
| $\mathrm{P}(1)-\mathrm{O}(4)$ | 1.513 |  | 1.520 |  | 1.506 |  |
| Average | 1.541 |  | 1.541 |  | 1.541 |  |
| $\mathrm{O}(1)-\mathrm{O}(2)$ | $2.533 \AA$ | $107.9^{\circ}$ | $2.533 \AA$ | $108 \cdot{ }^{\circ}$ | $2.534 \AA$ | $107.8^{\circ}$ |
| $\mathrm{O}(1)-\mathrm{O}(3)$ | 2.534 | 109.0 | 2.524 | 108.4 | 2.543 | 109.7 |
| $\mathrm{O}(1)-\mathrm{O}(4)$ | 2.493 | $106 \cdot 6$ | 2.482 | 105.4 | $2 \cdot 504$ | 107.9 |
| $\mathrm{O}(2)-\mathrm{O}(3)$ | 2.522 | 111.3 | 2.532 | 112.6 | 2.512 | $110 \cdot 1$ |
| $\mathrm{O}(2)-\mathrm{O}(4)$ | 2.465 | 107.8 | 2.472 | 108.4 | 2.456 | 107.2 |
| $\mathrm{O}(3)-\mathrm{O}(4)$ | 2.538 | 113.9 | 2.539 | 113.7 | 2.538 | 114.0 |
| Average | 2.514 | 109.4 | 2.514 | 109.4 | 2.515 | 109.4 |
| $\mathrm{P}(2)-\mathrm{O}(5)$ | $1.519 \AA$ |  | $1.520 \AA$ |  | $1.519 \AA$ |  |
| $\mathrm{P}(2)-\mathrm{O}(6)^{\mathrm{iv}}$ | 1.546 |  | 1.549 |  | 1.545 |  |
| $\mathrm{P}(2)-\mathrm{O}(7)^{\text {iv }}$ | 1.552 |  | 1.539 |  | 1.567 |  |
| $\mathrm{P}(2)-\mathrm{O}$ (8) | 1.521 |  | 1.522 |  | 1.523 |  |
| Average | 1.535 |  | 1.533 |  | 1.539 |  |
| $\mathrm{O}(5)-\mathrm{O}(6)^{\mathrm{iv}}$ | $2.519 \AA$ | $110 \cdot 6^{\circ}$ | 2.519 § | $110.3^{\circ}$ | $2.524 \AA$ | $111.0^{\circ}$ |
| $\mathrm{O}(5)-\mathrm{O}(7)^{\mathrm{iv}}$ | 2.511 | 109.7 | $2 \cdot 505$ | 110.0 | 2.520 | 109.5 |
| $\mathrm{O}(5)-\mathrm{O}(8)$ | $2 \cdot 500$ | $110 \cdot 7$ | $2 \cdot 507$ | 111.0 | 2.496 | $110 \cdot 3$ |
| $\mathrm{O}(6)^{\mathrm{iv}}-\mathrm{O}(7)^{\mathrm{iv}}$ | 2.451 | 104.6 | 2.458 | $105 \cdot 5$ | 2.446 | $103 \cdot 6$ |
| $\mathrm{O}(6)^{\mathrm{iv}}-\mathrm{O}(8)$ | $2 \cdot 520$ | 110.5 | 2.544 | 111.9 | 2.501 | 109.3 |
| $\mathrm{O}(7)^{\mathrm{iv}}-\mathrm{O}(8)$ | 2.525 | 110.5 | 2.475 | 108.0 | 2.575 | 112.9 |
| Average | $2 \cdot 504$ | 109.4 | 2.501 | 109.4 | $2 \cdot 510$ | 109.4 |

$\mathrm{H}(3), 1 \cdot 107(5) \AA$, seems to fit the corresponding $\mathrm{O} \cdots \mathrm{O}$ contact of 2.563 (2) $\AA$ better than in the $P \mathrm{I}$ model. With respect to the $\mathrm{Ca}-\mathrm{O}$ bonds (Table 3), only $\mathrm{Ca}(1)-\mathrm{O}(7)$ and $\mathrm{Ca}\left(1^{\prime}\right)-\mathrm{O}\left(7^{\prime}\right)$ do not conform clearly to the requirement for O bonded to H atoms.

## Choice of the model

In order to interpret the results of diffraction experiments, a few other chemical-physical properties of monetite should be taken into account.

A study of the heat capacity vs temperature curve (Egan \& Wakefield, 1964) showed that it had a jump between a maximum at 273 K and a minimum at 287 $K$; shifts of a few degrees for these values were observed in different samples. Such an anomaly could approximate a discontinuous change of heat capacity involving a second-order phase transition; e.g. an order/disorder transformation where only H positions would be significantly affected, since no variations of the X-ray powder spectrum were detected. The phase

Table 3. Interatomic distances in the Ca coordination polyhedra

For explanation and e.s.d.'s see Table 2.

|  | PĪ | P1 |  |
| :---: | :---: | :---: | :---: |
| $\mathrm{Ca}(1)-\mathrm{O}(1)^{\text {i }}$ | $2.419 \AA$ | $2.415 \AA$ | $2.424 \AA$ |
| $\mathrm{Ca}(1)-\mathrm{O}(2)$ | 2.447 | 2.448 | 2.447 |
| $\mathrm{Ca}(1)-\mathrm{O}(6)$ | 2.448 | 2.481 | 2.417 |
| $\mathrm{Ca}(1)-\mathrm{O}(7)$ | 2.764 | 2.794 | 2.731 |
| $\mathrm{Ca}(1)-\mathrm{O}(8)$ | 2.370 | 2.321 | 2.420 |
| $\mathrm{Ca}(1)-\mathrm{O}\left(2^{\prime}\right)^{i \mathrm{ii}}$ | 2.425 | 2.430 | 2.421 |
| $\mathrm{Ca}(1)-\mathrm{O}\left(4^{\prime}\right)^{\text {iii }}$ | 2.293 | 2.286 | 2.301 |
| Average | 2.452 | 2.454 | 2.452 |
| $\mathrm{Ca}(2)-\mathrm{O}(2)$. | 2.575 | 2.569 | 2.579 |
| $\mathrm{Ca}(2)-\mathrm{O}(3)^{\mathrm{j}}$ | 2.374 | 2.359 | 2.389 |
| $\mathrm{Ca}(2)-\mathrm{O}(4)$ | 2.482 | 2.496 | 2.467 |
| $\mathrm{Ca}(2)-\mathrm{O}(5)^{v}$ | 2.435 | 2.432 | 2.445 |
| $\mathrm{Ca}(2)-\mathrm{O}(8)$ | 2.478 | 2.482 | 2.472 |
| $\mathrm{Ca}(2)-\mathrm{O}\left(3^{\prime}\right)^{\text {iii }}$ | 2.485 | 2.478 | 2.488 |
| $\mathrm{Ca}(2)-\mathrm{O}\left(6^{\prime}\right)^{\text {ii }}$ | 2.518 | 2.499 | 2.536 |
| $\mathrm{Ca}(2)-\mathrm{O}\left(7^{\prime}\right)^{\text {iii }}$ | 2.503 | 2.521 | 2.489 |
| Average | 2.481 | 2.480 | 2.483 |

transition seems to be supported by approximate measurements of the dielectric constant in the range 300 to 220 K : indeed, after a rapid decrease between 300 and 270 K , it becomes stable at lower temperatures.

Besides the already mentioned piezoelectricity tests (Denne \& Jones, 1971), new measurements, based on the second optical harmonic generation by non-centrosymmetric crystals, were performed by a Nd laser with $\lambda=1.06 \mu \mathrm{~m}$ at the Centro di Studio per la Cristallografia Strutturale del CNR (Pavia); a very small signal, yet significantly above the noise, was detected.

The following explanation could account for all the experimental results: monetite shows a low-temperature ordered phase and a high-temperature disordered phase, which correspond substantially to the $P 1$ and $P \overline{1}$ structural models discussed above. The diffraction data were collected at room temperature, just at the upper end of the phase transition, when some fractions of the crystals were still in the ordered arrangement. In this light, most features of the problem are interpretable: (i) the successful $P \overline{1}$ refinement; (ii) some crystalchemical inconsistencies of the ordered $P 1$ refinement; (iii) feeble physical effects characteristic of non-centrosymmetry. Structurally, then, the presence of a small quantity of $P 1$ phase seems to be detected better by the displacement of $\mathbf{H}(1)$ off the centre than by the ordering of $\mathrm{H}(3)$.

It should be added that in both $P \overline{\mathrm{I}}$ and $P 1$ final difference maps the two largest negative peaks ( -14 and $-8 \times 10^{-14} \mathrm{~cm} \AA^{-3}$ ) appeared in a cavity delimited by $\mathrm{O}(6), \mathrm{O}(8), \mathrm{O}\left(6^{\prime}\right)$ and $\mathrm{O}\left(8^{\prime}\right)$; they lay in the centre of the cavity, at 1.59 and $2.05 \AA$ from $O(6)$ and $O(8)$, and on the line $\mathrm{O}(6) \cdots \mathrm{O}\left(8^{\prime}\right)$ at $1.01 \AA$ from $\mathrm{O}\left(8^{\prime}\right)$. These peaks could account for $10 \%$ of a proton approximately, and their positions might be involved in the disorder of $\mathrm{H}(3)$, in view of the reduced height of the corresponding peak observed on Fourier maps.

## KKM effect and thermal motion

From the heavy-atom parameters and the observed structure factors published in the X-ray work (Dickens

Table 4. Interatomic distances $(\AA)$ and angles $\left({ }^{\circ}\right)$ in the hydrogen bonds
For each set of bonds the first line refers to the $P \overline{1}$ model, the second and third lines refer to the corresponding and to the pseudocentrosymmetric bond in the $P 1$ model respectively

| $\begin{array}{llll}A & B & C & D\end{array}$ | $B C$ | $C D$ | $B D$ | $\angle A B C$ | $\angle B C D$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(1)-\mathrm{O}(1)-\mathrm{H}(2) \cdots \mathrm{O}\left(5^{\prime}\right)^{\mathrm{ii}}$ | 1.042 (2) | 1.521 (2) | 2.560 (1) | 117.0(1) | 175.6 (1) |
|  | 1.057 (3) | 1.515 (3) | 2.568 (2) | 117.3 (2) | 174.8 (3) |
|  | 1.027 (3) | 1.529 (3) | 2.553 (2) | 116.9 (2) | 175.3 (2) |
| $\mathrm{P}(2)^{\text {vi }}-\mathrm{O}(6)-\mathrm{H}(3) \cdots \mathrm{O}\left(8^{\prime}\right)^{\text {ii }}$ | 1.153 (6) | 1.627 (6) | 2.658 (1) | 119.3 (3) | 145.5 (5) |
|  | $1 \cdot 107$ (5) | 1.560 (5) | 2.563 (2) | 122.1 (3) | 147.4 (4) |
|  | - | - | 2.753 (2) | - | - |
| $\mathrm{P}(2)^{\text {vi }}-\mathrm{O}(7)-\mathrm{H}(1)-\mathrm{O}\left(7^{\prime}\right)$ | 1.230 (1) | 1.230 (1) | 2.459 (1) | 108.1 (1) | 180 |
|  | 1.283 (3) | 1.182 (3) | 2.461 (2) | - | 173.5 (3) |
|  | 1.182 (3) | 1.283 (3) | 2.461 (2) | 106.7(2) | 173.5 (3) |

et al., 1972), a calculation of $F_{c}$ gave $R=0.037$; the scale factor obtained was 1.024 , and secondary extinction and anomalous scattering were not taken into account. The syntheses $\Delta F_{X}, \Delta F_{N}$ and $F_{X}-F_{N}$ were then calculated in space group $P \overline{1}$, omitting the contribution of all H atoms for $\Delta F_{X}$ and of $\mathrm{H}(1)$ only for $\Delta F_{N}$ : the corresponding maps about the origin are shown in Fig. 2(a), (b) and (c). According to Dickens et al. (1972), $\mathrm{H}(1)$ was located as a single peak at the origin $0.77 \mathrm{e} \AA^{-3}$ high; the different height in our $\Delta F_{X}$ calculation can be explained by a change of scale factor: the multiple peaks may have been missed by the above authors through use of a coarse grid.


Fig. 2. Difference synthesis maps in the plane determined by the $c$ axis and the $\mathrm{O}(7) \cdots \mathrm{O}\left(7^{\prime}\right)$ direction (dashed line). Contours are
 $\AA^{-3}$ for (a) $\Delta F_{X}$, (b) $\Delta F_{N}$ and (c) $F_{X}-F_{N}$ respectively; zero and negative lines are broken. Distances and angles relevant to the maxima $1^{\prime}$ and $1^{\prime \prime}$ in (a) are: $O(7) \cdots\left(1^{\prime}\right)=0 \cdot 68,\left(1^{\prime}\right) \cdots O\left(7^{\prime}\right)=$ $1 \cdot 81,\left(1^{\prime}\right) \cdots\left(1^{\prime \prime}\right)=1 \cdot 19 \AA, \angle \mathrm{O}(7)-\left(1^{\prime}\right) \cdots \mathrm{O}\left(7^{\prime}\right)=161^{\circ}$.

The peculiar aspects of Fig. 2 seem to be interpretable as a particular version of the KKM effect (MacDonald, Speakman \& Hadži, 1972), differing both from the ordinary twin-peaks case in $\Delta F_{X}$ (Currie, Speakman, Kanters \& Kroon, 1975; Ferraris \& Franchini-Angela, 1973) and from the case found by MacDonald et al. (1972), with normal $\Delta F_{X}$ and $F_{X}-F_{N}$ showing multiple peaks. The unusual presence of three peaks in the $\Delta F_{X}$ map could be accounted for by the mixing of ordered and disordered phases discussed above. Probably, separate peaks can be detected in electronic but not in nuclear density maps because of electron polarizability effects (Catti \& Ferraris, 1976).

The two ellipsoids representing the total motion and the difference motion [with respect to $\mathrm{O}(7)$ ] of $\mathrm{H}(1)$ were compared; both show their directions of maximum displacement within about $10^{\circ}$ of the $\mathrm{O}(7) \cdots$ $O\left(7^{\prime}\right)$ line. Such an effect probably should be related to the structural disorder, in this case, rather than to the shape of the potential well as in very short hydrogen bonds not suspected of disorder (Speakman, 1975). The root-mean-square displacements along principal directions of thermal vibration for all atoms (Table 5) are slightly smaller in the $P 1$ refinement, on average, than in $P \overline{1}$, particularly for atoms which are involved directly in the order/disorder process.

## Comparison of the $X$-ray and neutron parameters

Heavy-atom parameters from the $P \overline{1}$ neutron refinement and from the X-ray refinement (Dickens et al., 1972) were compared statistically (Abrahams, 1974) through $\chi^{2}$ tests (Table 6) and half- or full-normal

Table 5. Root-mean-square displacements $\left(\AA \times 10^{3}\right)$ along the principal directions of vibration

|  | $P \overline{1}$ | $P 1$ |  |  | $P \overline{1}$ | $P 1$ |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $\mathrm{P}(1)$ | 77 | 72 | 70 | $\mathrm{O}(5)$ | 104 | 105 | 100 |
|  | 83 | 82 | 82 |  | 169 | 158 | 163 |
|  | 70 | 67 | 67 |  | 83 | 76 | 81 |
| $\mathrm{P}(2)$ | 90 | 93 | 85 | $\mathrm{O}(6)$ | 126 | 123 | 126 |
|  | 111 | 111 | 109 |  | 173 | 172 | 163 |
|  | 82 | 75 | 83 |  | 88 | 84 | 86 |
| $\mathrm{Ca}(1)$ | 97 | 95 | 94 | $\mathrm{O}(7)$ | 100 | 97 | 99 |
|  | 139 | 134 | 139 |  | 148 | 145 | 142 |
|  | 83 | 83 | 78 |  | 83 | 76 | 78 |
| $\mathrm{Ca}(2)$ | 103 | 99 | 101 | $\mathrm{O}(8)$ | 118 | 114 | 117 |
|  | 130 | 127 | 128 |  | 194 | 171 | 172 |
|  | 82 | 80 | 78 |  | 88 | 91 | 78 |
| $\mathrm{O}(1)$ | 106 | 102 | 106 | $\mathrm{H}(1)$ | 181 | 171 |  |
|  | 129 | 126 | 128 |  | 215 | 210 |  |
| $\mathrm{O}(2)$ | 80 | 78 | 81 |  | 130 | 125 |  |
|  | 102 | 101 | 103 | $\mathrm{H}(2)$ | 171 | 169 | 168 |
|  | 114 | 114 | 110 |  | 175 | 175 | 176 |
| $\mathrm{O}(3)$ | 74 | 69 | 70 |  | 135 | 133 | 134 |
|  | 109 | 109 | 106 | $\mathrm{H}(3)$ | 197 | 207 |  |
|  | 121 | 114 | 119 |  | 353 | 343 |  |
| $\mathrm{O}(4)$ | 77 | 74 | 76 |  | 161 | 159 |  |
|  | 104 | 102 | 101 |  |  |  |  |
|  | 130 | 127 | 127 |  |  |  |  |
|  | 75 | 73 | 68 |  |  |  |  |

Table 6. $\chi^{2}$ analysis for sets of structural parameters with $v$ degrees of freedom; $\delta p_{\max }$ is the maximum value of the weighted $p_{X}-p_{N}$ difference for the set

|  | $v$ | $\delta p_{\text {max }}$ | $\chi^{2}$ | $\chi \chi_{\nu, 0.05}^{2}$ |
| :---: | :---: | :---: | :---: | :---: |
| $x$ | 12 | $5 \cdot 7$ | 98.7 |  |
| $y$ | 12 | 14.7 | $830 \cdot 1$ |  |
| $z$ | 12 | $16 \cdot 3$ | $680 \cdot 2$ |  |
| $B_{11}$ | 12 | $6 \cdot 5$ | 184.8 |  |
| $B_{22}$ | 12 | $6 \cdot 7$ | $63 \cdot 6$ | 21 |
| $B_{33}$ | 12 | $5 \cdot 7$ | $166 \cdot 0$ |  |
| $B_{12}$ | 12 | $4 \cdot 2$ | 44.3 |  |
| $B_{13}$ | 12 | $7 \cdot 8$ | 108.8 |  |
| $B_{23}$ | 12 | $2 \cdot 1$ | 11.9 |  |
| $B_{i i}$ | 36 | $6 \cdot 7$ | 414.4 |  |
| $B_{i j}$ | 36 | $7 \cdot 8$ | $265 \cdot 0\}$ | 51 |
| All coordinates | 36 | $16 \cdot 3$ | 1609 | 5 |
| All $B_{i j}$ | 72 | 7.8 | 679.4 | 92 |
| All parameters | 108 | $16 \cdot 3$ | 2288 | 133 |
| $\mathrm{P}(1)-\mathrm{O}$ | 4 | 1.8 | $7.5\}$ | 9.5 |
| $\mathrm{P}(2)-\mathrm{O}$ | 4 | $6 \cdot 0$ | 57.9 $\}$ | 9.5 |
| $\mathrm{Ca}(1) \cdots \mathrm{O}$ | 7 | 4.9 | $45 \cdot 1$ | 14.4 |
| $\mathrm{Ca}(2) \cdots \mathrm{O}$ | 8 | $4 \cdot 8$ | $79 \cdot 2$ | 15.5 |

probability plots (not reported); the second kind of plot was used to analyse $B_{11}, B_{22}$ and $B_{33}$, since in this case the sign of the difference is relevant. The results of the two experiments appear to differ significantly, and this should not be due to a low reliability of the e.s.d.'s only, even if they are probably underestimated, since all the experimental values lie above the ideal straight line. In particular, the plot fails to be linear for positional parameters, and it fails to intersect the origin for diagonal thermal parameters. The latter effect is caused by $B_{i i}$ values being larger for X-ray data ( 27 out of 36 ), and has been related (Hamilton, 1969) to the inadequacy of the spherical approximation for X-ray scattering factors. The significant difference between positional parameters from the two experiments (confirmed by comparing the $\mathrm{P}-\mathrm{O}$ and $\mathrm{Ca}-\mathrm{O}$ distances) could be accounted for by assuming that the measurements were performed on two samples with different amounts of order/disorder. Actually, the largest differences are shown by $O(6), O(7)$ and $O(8)$, which are involved in the disorder of the H atoms, while the atoms of the $\mathrm{P}(1)$ tetrahedron behave more regularly.

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[^0]:    * Paper presented at the Tenth International Congress of Crystallography, 7-15 August 1975, Amsterdam.
    $\dagger$ Atoms are numbered according to Denne \& Jones (1971); Dickens et al. (1972) interchanged $\mathrm{H}(1)$ and $\mathrm{H}(2)$. A prime denotes an atom in a centrosymmetric (or quasi-centrosymmetric) position for the $P \overline{1}$ (or $P 1$ ) model. Roman numerals represent the translations: (i) $-\mathbf{c}$; (ii) $+\mathbf{a}+\mathbf{b}+\mathbf{c}$; (iii) $+\mathbf{b}+\mathbf{c}$; (iv) $+\mathbf{b}$; (v) $+\mathbf{c}$; (vi) $-\mathbf{b}$; (vii) $+\mathbf{a}$ $+\mathbf{c}$.

